Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____
I declare this is my own work.

A-level
MATHEMATICS

Paper 2

Tuesday 13 June 2023 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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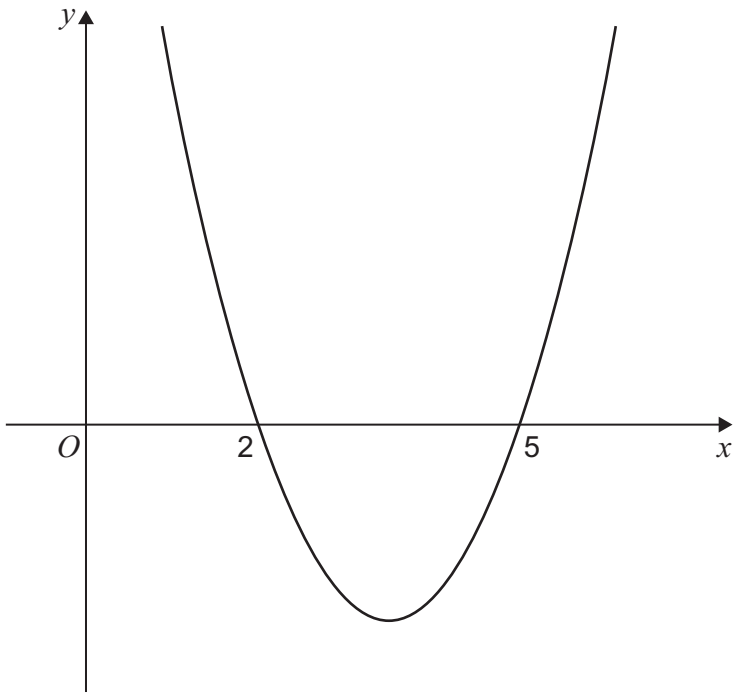
Section A

Answer **all** questions in the spaces provided.

1 The graph of $y = ax^2 + bx + c$ has roots $x = 2$ and $x = 5$, as shown in the diagram below.



State the set of values of x which satisfy

$$ax^2 + bx + c > 0$$

Tick (✓) **one** box.

[1 mark]

$\{x : x < 2\} \cup \{x : x > 5\}$ ☐

$\{x : 0 < x < 2\} \cap \{x : x > 5\}$ ☐

$\{x : 2 < x < 5\}$ ☐

$\{x : 2 > x > 5\}$ ☐

Do not write outside the box

2 It is given that

$$\int_0^6 f(x)\,dx = 20 \quad \text{and} \quad \int_3^6 f(x)\,dx = -10$$

Find the value of $\int_0^3 f(x)\,dx$

Circle your answer.

[1 mark]

-30 $\quad$ -10 $\quad$ 10 $\quad$ 30

3 A circle has equation

$$(x-5)^2 + (y-13)^2 = 16$$

Find the radius of the circle.

Circle your answer.

[1 mark]

4 $\quad$ 12 $\quad$ 16 $\quad$ 256

Turn over for the next question

Do not write outside the box

4 A curve has equation

$$y = \frac{x^2}{8} + 4\sqrt{x}$$

4 (a) Find an expression for $\frac{dy}{dx}$

[3 marks]

4 (b) The point P with coordinates (4, 10) lies on the curve.

Find an equation of the tangent to the curve at the point P

[2 marks]

Do not write outside the box

4 (c) Show that the curve has no stationary points.

[2 marks]

Turn over for the next question

Do not write outside the box

5 Ziad is training to become a long-distance swimmer.

He trains every day by swimming lengths at his local pool.

The length of the pool is 25 metres.

Each day he increases the number of lengths that he swims by four.

On his first day of training, Ziad swims 10 lengths of the pool.

5 (a) Write down an expression for the number of lengths Ziad will swim on his nth day of training.

[1 mark]

5 (b) (i) Ziad's target is to be able to swim at least 3000 metres in one day.

Determine the minimum number of days he will need to train to reach his target.

[3 marks]

Do not write outside the box

5 (b) (ii) Ziad's coach claims that when he reaches his target he will have covered a total distance of over 50 000 metres.

Determine if Ziad's coach is correct.

[3 marks]

Turn over for the next question

Do not write outside the box

6 Victoria, a market researcher, believes the average weekly value, £ V million, of online grocery sales in the UK has grown exponentially since 2009.

Victoria models the incomplete data, shown in the table, using the formula

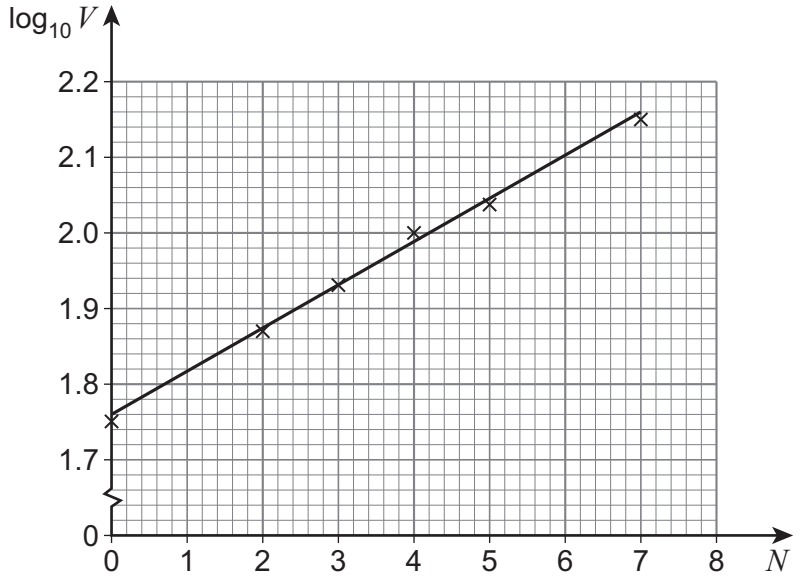
$$V = a \times b^N$$

where N is the number of years since 2009 and a and b are constants.

Year	2009	2010	2011	2012	2013	2014	2015	2016
Average Weekly Sales £ V million	56.4		74.5	86.9	97.7	109.3		141.9

6 (a) Victoria wishes to determine the values of a and b in her formula.

To do this she plots a graph of $\log_{10} V$ against N and then draws a line of best fit as shown in the diagram below.



The equation of Victoria's line of best fit is

$$\log_{10} V = 0.057N + 1.76$$

6 (a) (i) Use the equation of Victoria's line of best fit to show that, correct to three significant figures, $a = 57.5$

[1 mark]

Do not write outside the box

6 (a) (ii) Use the equation of Victoria's line of best fit to find the value of b

Give your answer to three significant figures.

[1 mark]

6 (b) According to Victoria's model, state the yearly percentage increase in the average weekly value of online grocery sales.

[1 mark]

6 (c) (i) Use Victoria's model to predict the average weekly value of online grocery sales in 2025.

[2 marks]

6 (c) (ii) Explain why the prediction made in part **(c)(i)** may be unreliable.

[1 mark]

Do not write outside the box

7 The functions f and g are defined by

$$\mathrm{f}(x) = \sqrt{10 - 2x} \quad \text{for} \quad x \leq 5$$

$$\mathrm{g}(x) = \frac{1}{x} \quad \text{for} \quad x \neq 0$$

The function h has maximum possible domain and is defined by

$$\mathrm{h}(x) = \mathrm{gf}(x)$$

7 (a) Find an expression for $\mathrm{h}(x)$

[1 mark]

7 (b) Find the domain of h

[1 mark]

7 (c) Show that $\mathrm{h}^{-1}(x) = 5 - \dfrac{1}{2x^2}$

[3 marks]

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Turn over for the next question

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ANSWER IN THE SPACES PROVIDED

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8 (a) Given that $\cos\theta \neq \pm 1$, prove the identity

$$\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} \equiv 2\operatorname{cosec}^2\theta$$

[4 marks]

8 (b) Hence, find the set of values of A for which the equation

$$\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} = A$$

has real solutions.

Fully justify your answer.

[3 marks]

Do not write outside the box

8 (c) Given that θ is obtuse and

$$\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}=16$$

find the exact value of $\cot\theta$

[3 marks]

Turn over for the next question

Do not write outside the box

9 (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(1+x)^{-\frac{1}{2}}$$

[2 marks]

9 (b) A student substitutes $x = 2$ into the expansion of $(1+x)^{-\frac{1}{2}}$ to find an approximation for $\frac{1}{\sqrt{3}}$

Explain the mistake in the student's approach.

[1 mark]

Do not write outside the box

9 (c) By substituting $x = -\frac{1}{4}$ in your expansion for $(1+x)^{-\frac{1}{2}}$ find an approximation for $\frac{1}{\sqrt{3}}$

Give your answer to three significant figures.

[3 marks]

Turn over for the next question

Do not write outside the box

10 (a) Expand and simplify $(a - b)^2$

[1 mark]

10 (b) Peter thinks that the sum of any rational number and its reciprocal is always greater than 2

Peter checks two examples:

$$\frac{2}{3} + \frac{3}{2} = 2.1\dot{6}$$

$$2 + \frac{1}{2} = 2.5$$

Use a counter example to show that Peter is **incorrect**.

[2 marks]

Do not write outside the box

10 (c) Given that a and b are distinct positive numbers, use proof by contradiction to prove that

$$\frac{a}{b} + \frac{b}{a} > 2$$

[3 marks]

END OF SECTION A

TURN OVER FOR SECTION B

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Section B

Answer **all** questions in the spaces provided.

11 A decoration is hanging freely from a fixed point on a ceiling.

The decoration has a mass of 0.2 kilograms.

The decoration is hanging by a light, inextensible wire.

The wire is 0.1 metres long.

Find the tension in the wire.

Circle your answer.

[1 mark]

0.02 N 0.02g N 0.2 N 0.2g N

12 A particle moves in a straight line.

After the first 4 seconds of its motion, the displacement of the particle from its initial position is 0 metres.

One of the graphs on the opposite page shows the velocity $v\,\mathrm{m\,s^{-1}}$ of the particle after time t seconds of its motion.

Identify the correct graph.

Tick (✓) **one** box.

[1 mark]

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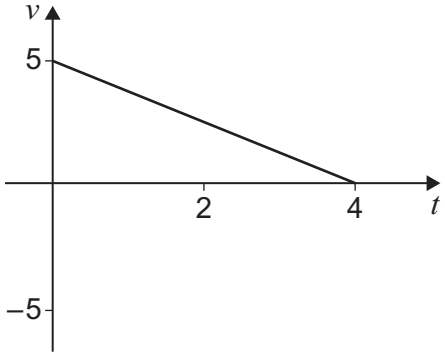

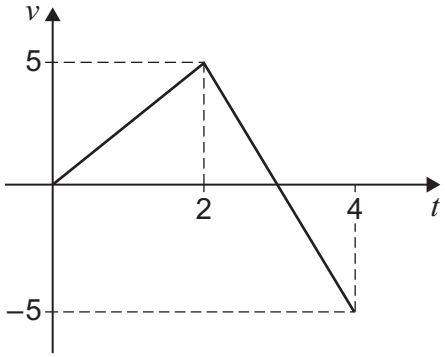

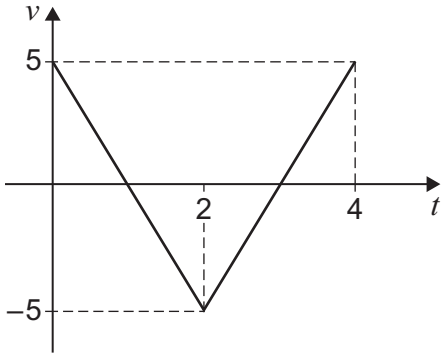

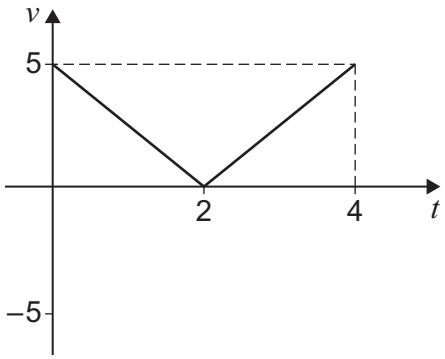

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13 A ball falls freely towards the Earth.

The ball passes through two different fixed points *M* and *N* before reaching the Earth's surface.

At *M* the ball has velocity $u\,\mathrm{m\,s^{-1}}$
At *N* the ball has velocity $3u\,\mathrm{m\,s^{-1}}$

It can be assumed that:

- the motion is due to gravitational force only
- the acceleration due to gravity remains constant throughout.

13 (a) Show that the time taken for the ball to travel from *M* to *N* is $\frac{2u}{g}$ seconds.

[2 marks]

Do not write outside the box

13 (b) Point M is h metres above the Earth.

Show that $h > \dfrac{4u^2}{g}$

Fully justify your answer.

[3 marks]

Turn over for the next question

Do not write outside the box

14 A car has an initial velocity of $1\,\text{m}\,\text{s}^{-1}$

The car is moving in a straight line.

The acceleration $a\,\text{m}\,\text{s}^{-2}$ of the car at time t seconds is given by

$$a = 3kt^2 - 2kt + 1$$

where k is a constant.

When $t = 3$ the car has a velocity of $10\,\text{m}\,\text{s}^{-1}$

Show that $k = \frac{1}{3}$

[4 marks]

Do not write outside the box

15 **In this question use $g = 9.8\,\text{m}\,\text{s}^{-2}$**

A particle, Q, moves in a straight line across a rough horizontal surface.

A horizontal driving force of magnitude D newtons acts on Q

Q moves with a constant acceleration of $0.91\,\text{m}\,\text{s}^{-2}$

Q has a **weight** of 0.65 N

The only resistance force acting on Q is due to friction.

The coefficient of friction between Q and the surface is 0.4

Find D

[4 marks]

Do not write outside the box

16 A particle moves under the action of two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$

It is given that

$$\mathbf{F}_1 = (1.6\mathbf{i} - 5\mathbf{j})\,\text{N}$$

$$\mathbf{F}_2 = (k\mathbf{i} + 5k\mathbf{j})\,\text{N}$$

where k is a constant.

The acceleration of the particle is $(3.2\mathbf{i} + 12\mathbf{j})\,\text{m}\,\text{s}^{-2}$

Find k

[4 marks]

Do not write outside the box

Turn over for the next question

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ANSWER IN THE SPACES PROVIDED

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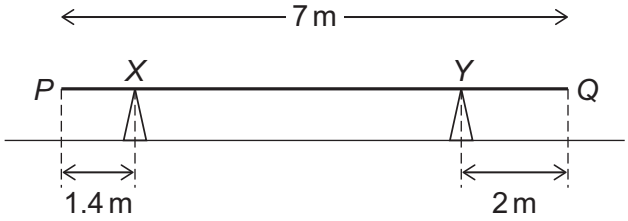
17 A uniform plank *PQ*, of length 7 metres, lies horizontally at rest, in equilibrium, on two fixed supports at points *X* and *Y*

The distance *PX* is 1.4 metres and the distance *QY* is 2 metres as shown in the diagram below.



17 (a) The reaction force on the plank at *X* is $4g$ newtons.

17 (a) (i) Show that the mass of the plank is 9.6 kilograms.

[2 marks]

17 (a) (ii) Find the reaction force, in terms of g, on the plank at *Y*

[2 marks]

Do not write outside the box

17 (b) The support at Y is moved so that the distance $QY = 1.4$ metres.

The plank remains horizontally at rest in equilibrium.

It is claimed that the reaction force at Y remains unchanged.

Explain, with a reason, whether this claim is correct.

[2 marks]

Turn over for the next question

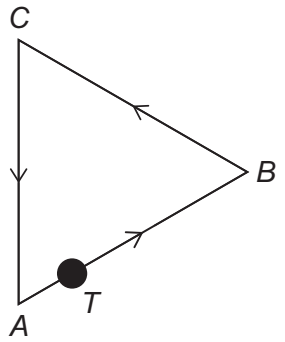
Do not write outside the box

18 In this question **i** and **j** are perpendicular unit vectors representing due east and due north respectively.

A particle, T, is moving on a plane at a constant speed.

The path followed by T makes the exact shape of a triangle ABC.

T moves around ABC in an anticlockwise direction as shown in the diagram below.



On its journey from A to B the velocity vector of T is $(3\mathbf{i} + \sqrt{3}\mathbf{j})\,\mathrm{m\,s^{-1}}$

18 (a) Find the speed of T as it moves from A to B

[1 mark]

18 (b) On its journey from B to C the velocity vector of T is $(-3\mathbf{i} + \sqrt{3}\mathbf{j})\,\mathrm{m\,s^{-1}}$

Show that the acute angle $ABC = 60°$

[2 marks]

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18 (c) It is given that *ABC* is an equilateral triangle.

T returns to its initial position after 9 seconds.

Vertex *B* lies at position vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ metres with respect to a fixed origin O

Find the position vector of *C*

[3 marks]

Turn over for the next question

Do not write outside the box

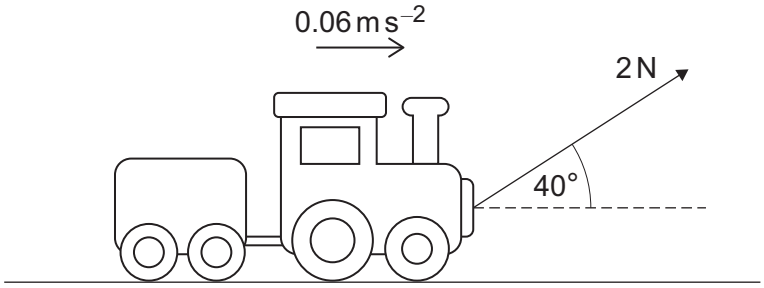
19 A wooden toy comprises a train engine and a trailer connected to each other by a light, inextensible rod.

The train engine has a mass of 1.5 kilograms.
The trailer has a mass 0.7 kilograms.

A string inclined at an angle of 40° above the horizontal is attached to the front of the train engine.

The tension in the string is 2 newtons.

As a result the toy moves forward, from rest, in a straight line along a horizontal surface with acceleration $0.06\,\text{m}\,\text{s}^{-2}$ as shown in the diagram below.



As it moves the train engine experiences a total resistance force of 0.8 N

19 (a) Show that the total resistance force experienced by the trailer is approximately 0.6 N

[4 marks]

Do not write outside the box

19 (b) At the instant that the toy reaches a speed of $0.5\,\mathrm{m\,s^{-1}}$ the string breaks.

As a result of this the train engine and trailer decelerate at a constant rate until they come to rest, having travelled a distance of h metres.

It can be assumed that the resistance forces remain unchanged.

19 (b) (i) Find the tension in the rod after the string has broken.

[4 marks]

Question 19 continues on the next page

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19 (b) (ii) Find h

[3 marks]

19 (c) State one modelling assumption that you have used about the rod when answering part **(b)(i)**.

[1 mark]

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Turn over for the next question

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ANSWER IN THE SPACES PROVIDED

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20 **In this question use $g = 9.8\,\text{m}\,\text{s}^{-2}$**

Nell and her pet dog Maia are visiting the beach.

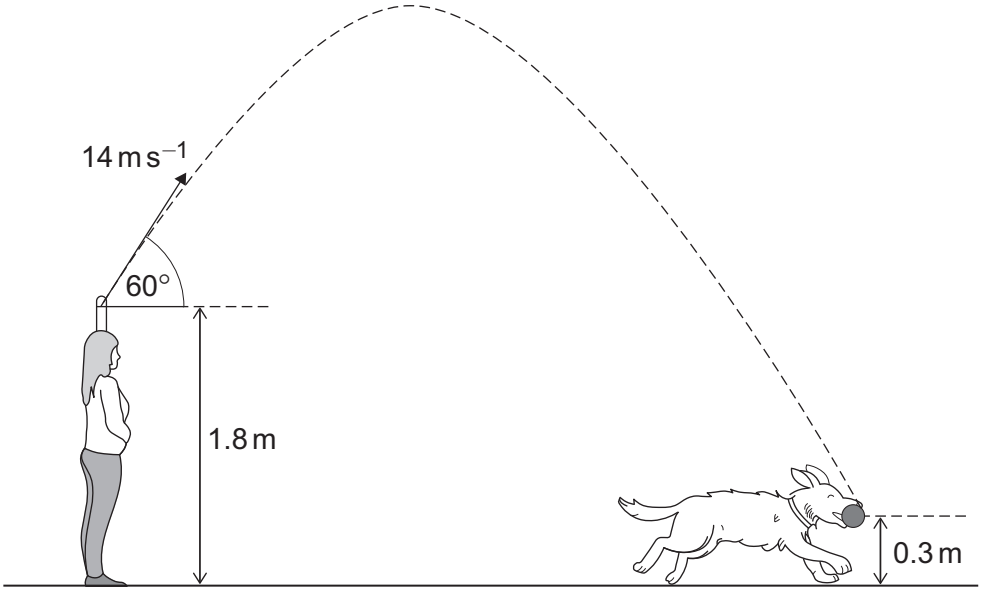
The beach surface can be assumed to be level and horizontal.

Nell and Maia are initially standing next to each other.

Nell throws a ball forward, from a height of 1.8 metres above the surface of the beach, at an angle of 60° above the horizontal with a speed of $14\,\text{m}\,\text{s}^{-1}$

Exactly 0.2 seconds **after** the ball is thrown, Maia sets off from Nell and runs across the surface of the beach, in a straight line with a constant acceleration $a\,\text{m}\,\text{s}^{-2}$

Maia catches the ball when it is 0.3 metres above ground level as shown in the diagram below.



Find a

[7 marks]

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END OF QUESTIONS

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